ISO/BIPM Guide: Uncertainty of measurement[1]

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Abstract

Starting from the general concept of a measuring process and the naive view of the measurement uncertainty, the paper presents the modern GUM view of the measurement uncertainty and discusses the interdependence of the standard measurement uncertainty and the expanded measurement uncertainty the last one being mostly used in trade and industry.

1 Introduction

The practice of measurement science has made us realize that the comparison of measured values requires, in addition to the proper value, a statement of the reliability and quality of that value. Such a statement will always be based on technical and/or scientific knowledge, i.e. on objective facts. Since a judgement is concerned, such a statement will nevertheless remain a subjective one. General acceptance of such a quality judgement can be achieved, however, if it is stated clearly in which way this judgement has been reached. This is the aim of the transparent uncertainty analysis.

2 The measurement process

The concept of the uncertainty of measurement was newly defined in 1992 by the ISO/BIPM "Guide to the expression of uncertainty in measurement", abbreviated to GUM [1]. It has meantime been internationally accepted as the basis for the determination of the measurement uncertainty[2] and has also been taken over by document EA-4/02 [2] of the "European co-operation for Accreditation" as

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[1] English version of a lecture given at the DGQ-VDI/VDE meeting in Langen/Hessen (1998) [12].

[2] The ISO/BIPM Guide is not a standard in the precise sense. However, it is the basis on which standards dealing with metrological subjects are to be drawn up. Its normative character is demonstrated by the support given by the following, widely recognized international institutions:

- **BIPM** - International Bureau of Weights and Measures,
- **OIML** - International Organization of Legal Metrology,
- **ISO** - International Organization for Standardization,
- **IEC** - International Electrotechnical Commission,
- **IUPAC** - International Union of Pure and Applied Chemistry,
- **IUPAP** - International Union of Pure and Applied Physics,
- **IFCC** - International Federation of Clinical Chemistry.

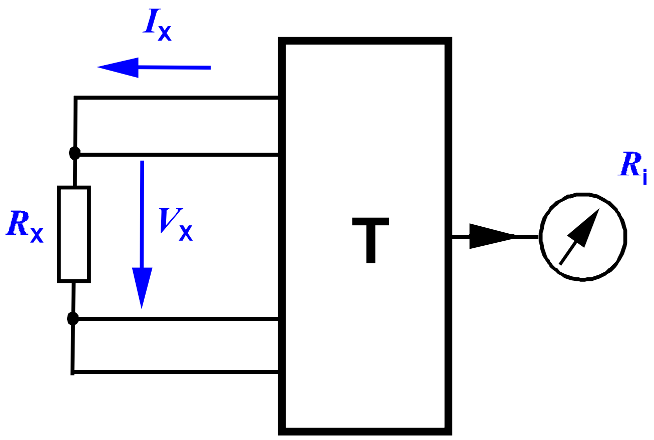
being mandatory for calibrations. The concept is based on characteristic features of the measurement process. The definitions of the terms are given in the VIM [3] and in the standard DIN 1319 [4].

It is the purpose of the measurement process to determine the value of a physical, measurable quantity X in relation to a unit. This fact is in general expressed by the following relation:

$$X = \{X\}[X] \qquad (1)$$

$\{X\}$ - value determined by the measurement,
$[X]$ - unit in relation to which the value is determined.

This equation shows that the essence of the measurement process is to assign a value by comparison: With the aid of the measurement process, the value of the quantity on the right hand side is transferred to the quantity to be measured, the measurand, given on the left hand side. This becomes quite clear, for example, when the resistance of a resistor is determined with a indicating instrument according to the four pole method as shown in Fig. 1.



$$R_X = (1 + d_1) \cdot (R_i + \Delta R_i) + d_2$$

Fig. 1 Determination of a resistance by an direct indicating instrument according to the four pole method.

R_X - resistance to be determined,
R_i - indicated resistance
ΔR_i - deviation, known from calibration, of the indicated value from the value that should be indicated (conventional true value). This deviation is usually referred to as the error of indication. If the indicating instrument has not been calibrated, this deviation is another contribution to the unknown deviations, that must be taken into account;
d_1 - unknown deviation from the ideal characteristic of the indicating instrument,
d_2 - unknown deviation due to the finite resolution of the indication and the null adjustment of the instrument.

A measurement - especially a more precise one - will in general not consist of such a simple assignment as shown in Fig. 1. Other influence quantities, than those depicted there, will play a role, such as the ambient temperature, the non-linearity of the transducer (in Fig. 1: the black box **T**) which transforms the measurand so that it can be indicated, the irregularity of the indicator scale, etc. However, this just means that the small unknown deviations d_1 and d_2 split further into sums of corresponding, small deviations. The basic structure of the physical relation shown in Fig. 1 will be preserved. On the whole, a general relation as shown by the diagram of Fig. 2 results for a measurement:

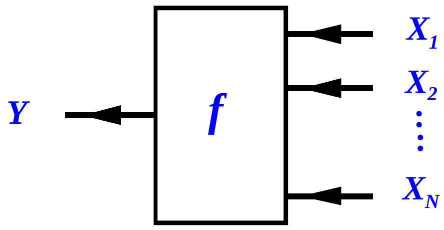


Fig. 2 Relation between the influence quantities and the measurand

There is a measurand Y whose value is to be determined from the values of the influence quantities $X_1, X_2, \ldots, X_N$. This relation is reflected by the physical relationship realized by the measurement process - here represented by the white box. In mathematical terms, the assignment in a measurement is made through the *model* of the evaluation

$$Y = f(X_1, X_2, \ldots, X_N) \qquad (2)$$

Y	-	quantity whose value is to be determined by measurement,
$X_1, X_2, \ldots, X_N$	-	influence quantities entering into the determination of the value to be measured,
N	-	number of influence quantities,
$f(X_1, X_2, \ldots, X_N)$	-	model function (mathematical relationship) by which the value of the measurand is determined from the influence quantities.

The left hand side symbolizes the measurand, the right hand side the mechanism according to which the quantity, whose value is assigned, is evaluated from the influence quantities.

3 The naive view: uncertainty interval

It is a basic experience of metrology that a measurement does not furnish an exact result. The result is rather affected by an uncertainty. This uncertainty is stated as the uncertainty of measurement which is to be defined in compliance with VIM (3.9), GUM (2.2.3) and DIN 1319-T1 (3.6):

The uncertainty of measurement is a parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could (according to the measurement performed) reasonably be attributed to the measurand.

The naive view of the measurement uncertainty regards the whole range of consistent values as an *uncertainty interval*, without weighting the values contained in it:

$$I_X = [x_-; x_+] \tag{3}$$

x_+, x_- - upper and lower bounds of the uncertainty interval.

Fig. 3 explains the relations.

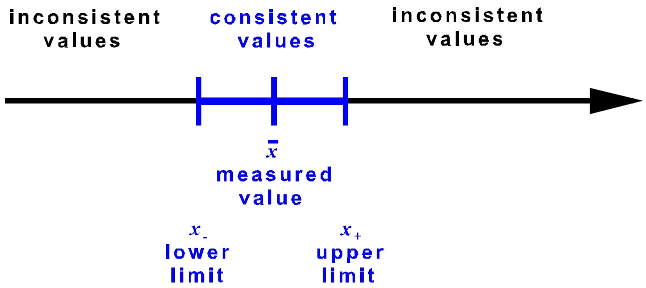


Fig. 3 Characteristic values of an uncertainty interval

The arithmetic mean value of the two bounds is considered to be the measured value

$$\bar{x} = \frac{x_+ + x_-}{2} \tag{4}$$

and the half-width of the uncertainty interval is regarded as the measure of the uncertainty

$$\Delta a = \frac{x_+ - x_-}{2} \tag{5}$$

This view of the measurement uncertainty is quite useful for measurements (e.g. Fig. 1) where the assignment is subject to only a few influence quantities. As the uncertainty of the measurand originates in the uncertainty of the influence quantities, the bounds for the result quantity must be calculated from the bounds of the input quantities using equation (2).

The equations for the arithmetic operations required for this purpose - addition, subtraction, multiplication and division - first look rather simple

$$\begin{aligned} I_{X_1} + I_{X_2} &= \left[(x_1 + x_2)_- ; (x_1 + x_2)_+\right] \\ I_{X_1} - I_{X_2} &= \left[(x_1 - x_2)_- ; (x_1 - x_2)_+\right] \\ I_{X_1} \cdot I_{X_2} &= \left[(x_1 \cdot x_2)_- ; (x_1 \cdot x_2)_+\right] \\ \frac{I_{X_1}}{I_{X_2}} &= \left[\left(\frac{x_1}{x_2}\right)_- ; \left(\frac{x_1}{x_2}\right)_+\right] \end{aligned} \tag{6}$$

When one enters into details of the calculation, equations result which comprise intricate permutations of the values and make necessary a number of case analyses which are ultimately rather complicated:

$$\begin{aligned} (x_1 + x_2)_- &= x_{1-} + x_{2-} \\ (x_1 + x_2)_+ &= x_{1+} + x_{2+} \end{aligned} \tag{6a}$$

$$\begin{aligned} (x_1 - x_2)_- &= x_{1-} - x_{2+} \\ (x_1 - x_2)_+ &= x_{1+} - x_{2-} \end{aligned} \tag{6b}$$

$$\begin{aligned} (x_1 \cdot x_2)_- &= \min(x_{1-}x_{2-}, x_{1+}x_{2-}, x_{1-}x_{2+}, x_{1+}x_{2+}) \\ (x_1 \cdot x_2)_+ &= \max(x_{1-}x_{2-}, x_{1+}x_{2-}, x_{1-}x_{2+}, x_{1+}x_{2+}) \end{aligned} \tag{6c}$$

$$\begin{aligned} \left(\frac{x_1}{x_2}\right)_- &= \min\left(\frac{x_{1-}}{x_{2-}}, \frac{x_{1+}}{x_{2-}}, \frac{x_{1-}}{x_{2+}}, \frac{x_{1+}}{x_{2+}}\right) \\ \left(\frac{x_1}{x_2}\right)_+ &= \max\left(\frac{x_{1-}}{x_{2-}}, \frac{x_{1+}}{x_{2-}}, \frac{x_{1-}}{x_{2+}}, \frac{x_{1+}}{x_{2+}}\right) \end{aligned} \tag{6d}$$

The naive view of the measurement uncertainty is, therefore, useful only for certain, rather rough classifications. It cannot be considered as a useful basis for a versatile quality measure. The calculational effort required is great, even if only a few influence quantities are involved, and it is hardly manageable for more complex mathematical expressions, e.g. for trigonometric functions, without additional decision rules concerning the bounds being established.

There is, however, one point whose problematic nature becomes fully apparent even if the naive view is adopted: proof of the conformity of a value with a specification. When the measurement uncertainty with an uncertainty interval is considered, a so-called range of uncertainty (or rather: a range of indifference) as shown in Fig. 4 results, in addition to the ranges of conformity and nonconformity. Clear conformity or nonconformity is given only if the measured value lies in the respective range. Values in the uncertainty range must either be judged more specifically [5] or be determined more precisely. The way of proceeding was discussed in detail within the framework program of MICROTECH 1996 [6]. The rules have been, respectively will be, laid down more precisely in ISO EN DIN 14253 [7].

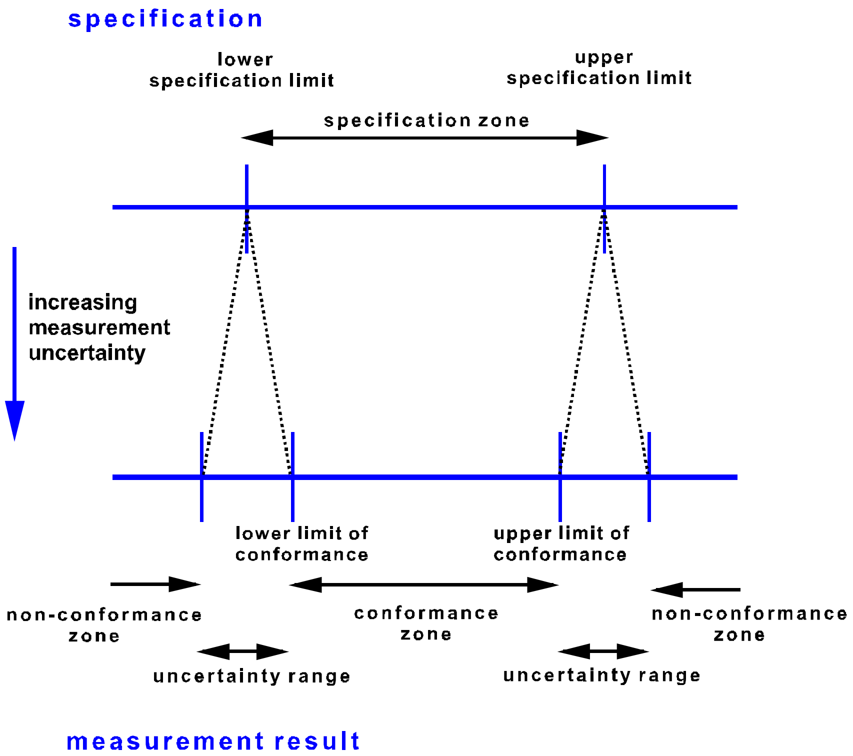


Fig. 4 Ranges of the conformity and nonconformity of a measured value with a specification, and ranges of uncertainty

A serious problem arises in the treatment of more precise measurements subject to many influence factors. Equations (6) furnish an uncertainty interval for the measurand, whose half-width is in most cases equal to the sum of the half-widths of the uncertainty intervals of the individual influence quantities. The half-widths thus become unrealistically large. An increase in precision, which always involves an increase in the number of influence quantities taken into account, does not lead to a substantial reduction of the measurand's uncertainty interval. The uncertainty range hardly becomes considerably narrower as a result of more precise measurements. With the naive view of the uncertainty of measurement, a more detailed examination of the measurement and its influence quantities is basically not very beneficial. However, the range within which a measured value conforms with a specification can be extended in two ways only: by re-evaluation of the specifications with an appropriate extension of the range of specification, or by a reduction of the uncertainty of measurement. A reduction of the uncertainty of measurement can be achieved only by improvement of the measuring facilities, which is usually costly and time-consuming, or by a more realistic judgement of the unknown errors of measurement as offered by the GUM.

4 The GUM view: standard uncertainty of measurement

The GUM introduces another view of the measurement uncertainty. Indication of an uncertainty interval is very crude. It leaves out of account that not all values consistent with the measurement conditions have the same chance of being achieved. It is usually known from special findings or fundamental considerations that values close to the centre of the uncertainty interval are more probable than values close to the limits. There are also cases where the situation is converse. The GUM assumes distributions or, more precisely, probability distributions, of the consistent values that reflect in mathematical form the knowledge about the conditions of measurement [8]. The measured value is the expectation value formed with the aid of the distribution; the measurement uncertainty associated with it is the standard deviation. It is obtained as the positive square root from the variance:

$$x = \mathrm{E}[X] \tag{7}$$

$$u(x) = \sqrt{\mathrm{Var}[X]} \tag{8}$$

x - measured value assigned to the quantity X,
$u(x)$ - standard uncertainty of measurement associated with the measured value x.

X symbolizes one of the quantities of the model relationship in equation (2), including the result quantity Y. The measurement uncertainty determined in this way is referred to as *standard uncertainty of measurement*. First, because it is the standard deviation of the distribution and, secondly, because it describes the mean width in such a fundamental way that it appears in many equations.

The naive view is to some extent also part of the GUM view. If the only thing one knows of the values of a quantity is that they lie between a lower bound a_- and an upper bound a_+, all values between these bounds are equal. This poor knowledge leads to the rectangular distribution represented in Fig. 5.

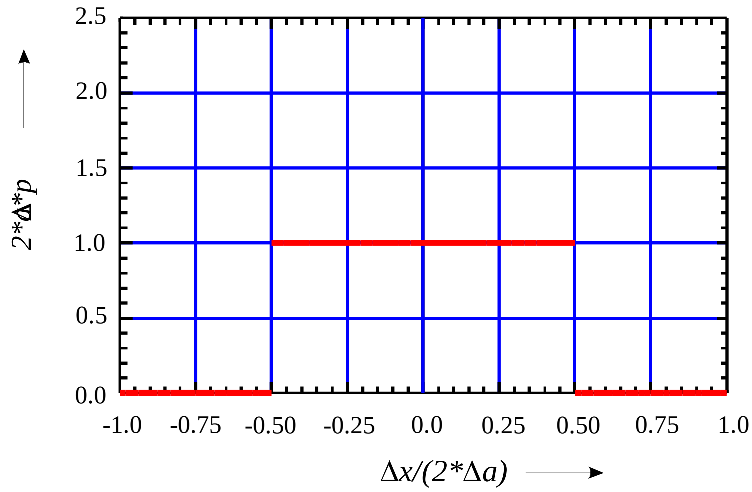


Fig. 5 Rectangular distribution - uniform distribution of the values between an upper and a lower bound.

The measured value is the same as in equation (4); the standard uncertainty of measurement associated with it, however, results from the half-width of the variability interval to be

$$u(x) = \frac{\Delta a}{\sqrt{3}} \tag{9}$$

The distribution of the values in digital displays will be assumed to be a rectangular distribution. Half the digital resolution (half the quantizing step) around the displayed value defines the upper and lower bounds at which the display jumps to the respective neighbouring value.

Switching over to distributions of the values has an important consequence. The distribution of the values of a sum or of the difference of two input quantities is concentrated more strongly around the value of the measurement result. Fig. 6 shows a triangular distribution resulting from the difference between two rectangular distributions of the same half-width. The model functions read as follows:

$$Y = X_1 \pm X_2 \tag{10}$$

with respective values of the measurement result

$$y = x_1 \pm x_2 \tag{11}$$

and the associated standard uncertainty of measurement

$$u(y) = \frac{\Delta a}{\sqrt{6}} \tag{12}$$

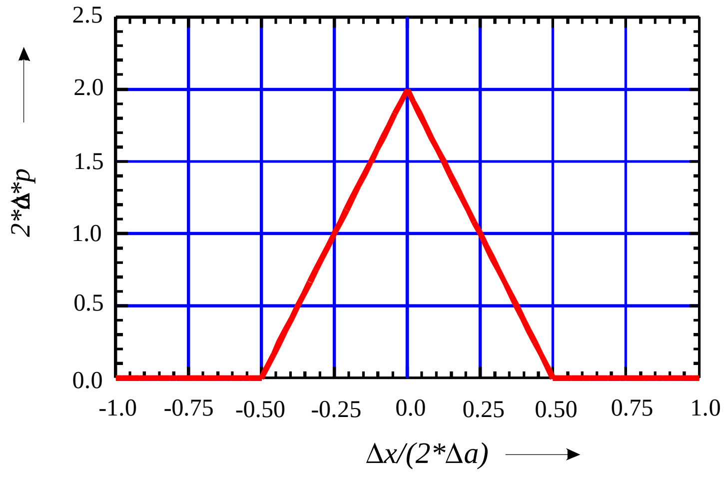


Fig. 6 Triangular distribution - resulting from a convolution of two rectangular distributions of the same half-width.

The value of the standard uncertainty of measurement is the same for the sum and the difference. The half-width in equation (12) is the half-width of the triangular distribution and amounts to twice the half-widths of the convoluted rectangular distributions. The curve shows that smaller deviations from the measured value are more probable than larger ones. The measured value can be determined from different combinations of the input values, whereas the limiting values can be determined from one combination only, namely the combination of the limiting values themselves.

For a general sum of three input quantities with rectangular distributions of identical half-width, almost a Gaussian bell-shaped error curve results as shown in Fig. 7.

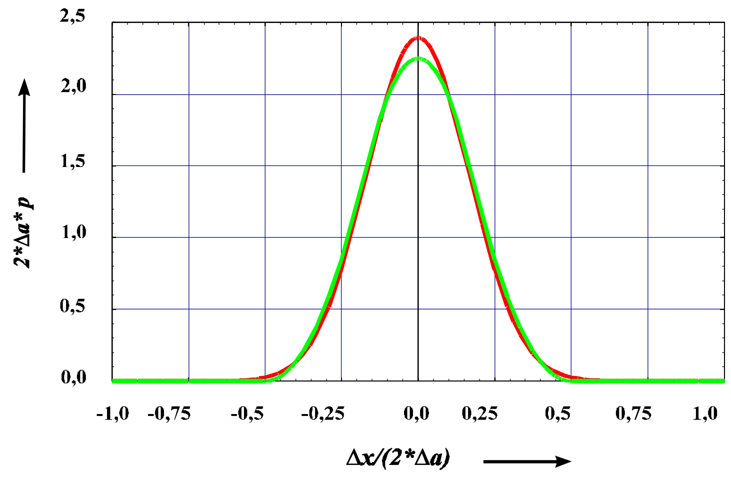


Fig. 7 Triply convoluted rectangular distribution in comparison with the normal distribution of equal variance.

The deviations from the normal distribution are smaller than 5% over the whole range. This reflects the effect of the Central Limit Theorem. This theorem states that the values of a sum quantity are normally distributed if the number of summands is sufficiently great and their variances, i.e. the squares of the standard uncertainties of measurement, do not differ too much. Three quantities are already sufficient in the case of rectangular distributions of equal half-width.

5 The GUM method

As the measurement uncertainty is always associated to a measured value, the analysis of the measurement uncertainty is inseparably linked with the determination of the measurement value. According to the GUM, the measurement value is obtained by inserting the values of the input quantities into the model function

$$y = f(x_1, x_2, \ldots, x_N) \qquad (13)$$

y - measured value, result of the measurement,
$x_1, x_2, \ldots, x_N$ input values, values of the influence quantities.

The square of the associated standard uncertainty of measurement is obtained as the sum of the squares of the *uncertainty contributions* of the individual input quantities

$$u(y) = \sqrt{u_1^2(y) + u_2^2(y) + \ldots + u_N^2(y)} \qquad (14)$$

$u_1(y), u_2(y), \ldots, u_N(y)$ - uncertainty contributions of the individual influence quantities.

The uncertainty contributions are calculated from the standard uncertainties of measurement associated to the input values, by multiplication by the *sensitivity coefficients*

$$u_i(y) = c_i u(x_i) \qquad (15)$$

c_i - sensitivity coefficient with respect to influence quantity X_i,.
$u(x_i)$ - standard uncertainty of measurement associated to the input value x_i.

The model is involved through the sensitivity coefficients. They are given by the partial derivatives of the model functions for the respective input quantities

$$c_i = \frac{\partial f}{\partial x_i} \qquad (16)$$

The sensitivity coefficients indicate to which extent small changes in the value of the respective input quantity affect the measured value. In the field of electrical engineering they are known under the designation 'steepness'. It will not be necessary in most cases to actually carry out the partial derivatives of equation (19). For two simple, but frequently occurring cases, the sensitivity coefficients can be directly read from the model function:

- If the model function is a general sum

$$y = p_1 x_1 + p_2 x_2 + \ldots + p_N x_N \quad (17)$$

$p_1, p_2, \ldots, p_N$ - numerical constants.

the sensitivity coefficients are identical with the numerical constants and the searched standard uncertainty of measurement is calculated according to

$$u(y) = \sqrt{p_1^2 u^2(x_1) + p_2^2 u^2(x_1) + \ldots + p_N^2 u^2(x_N)} \quad (18)$$

- If the model function is a general product

$$y = x_1^{p_1} \cdot x_2^{p_2} \cdot \ldots \cdot x_N^{p_N} \quad (19)$$

$p_1, p_2, \ldots, p_N$ - numerical constants.

the relative standard uncertainty of measurement

$$w(y) = \frac{u(y)}{y} \quad (20)$$

is the quantity suitable for the calculation. It is determined according to the equation

$$w(y) = \sqrt{p_1^2 w^2(x_1) + p_2^2 w^2(x_1) + \ldots + p_N^2 w^2(x_N)} \quad (21)$$

$w(x_1), w(x_2), \ldots, w(x_N)$ - relative standard uncertainty of measurement associated with the input values.

Bachmair demonstrated at the DGQ-VDI/VDE expert meeting concerning test equipment held at Langen (Hessen) [9] how the model construction of many metrological problems can be reduced to combinations of equations (17) to (21). His considerations which referred to the field of electricity can be easily transferred to other fields of metrology. For the remaining determination of the standard uncertainties of measurement associated with the input values, the GUM states two methods: the type A and the type B method of evaluation.

The type B method is to be applied if the complete value of an input quantity, i.e. the measured value and the associated measurement uncertainty, is known or if experience gained in the field of metrology allows a certain distribution to be assumed. In the first case, the measured value and the associated measurement uncertainty are directly given; they can be taken over. Calibration certificates state the associated expanded uncertainty of measurement. The standard uncertainty of measurement is obtained by dividing the expanded uncertainty by the coverage factor stated as well. If a certain distribution can be assumed as is the case with the resolution of a digital measuring instrument, the equations appropriate for the determination of the standard deviation are applied.

The type A method is to be applied if the input quantity, e.g. the quantity *Q*, is observed several times and different values are found. Evaluation is then made by statistical methods. The value is the arithmetic mean of the observations

$$\bar{q} = \frac{q_1 + q_2 + \ldots + q_n}{n} \qquad (22)$$

$q_1, q_2, \ldots, q_n$ - values observed
$u(x_i)$ - number of observations made.

The standard uncertainty of measurement to be associated is calculated by the equation

$$u(\bar{q}) = \sqrt{\frac{(q_1 - \bar{q})^2 + (q_2 - \bar{q})^2 + \ldots + (q_n - \bar{q})^2}{n(n-1)}} \qquad (23)$$

At least 10 observations should be made in order that the standard uncertainty of measurement determined in this way is a statistically reliable value.

Equations (13) to (23) form the closed system which is to be used for the determination of the measurement value and of the associated standard uncertainty of measurement. For routine evaluations, they can easily be transposed in spreadsheet programs. However, it is also possible to use evaluation programs more friendly to the user, which prepare user-guided uncertainty analyses, including reports[i]. A collection of analyses of measurement uncertainties is given as an example in supplement S1 and S2 to EA-4/02 [10,11]; further examples will be published in the next supplements to come..

The following example of the calibration of a standard resistor gives three notations, each in normal and relative terms, to be used to state the complete result of a measurement comprising measured value and associated standard uncertainty:

The measured resistance of the calibrated 10 Ω standard resistor at a temperature of 23 °C and a measuring current of 100 µA is

1*)	10 000,178 Ω; 0,0083 Ω	or	10 000,178 Ω; 0,83*10^{-6}
2*)	10 000,178 Ω (83 mΩ)	or	10 000,178 Ω (0,83*10^{-6})
3*)	(10 000,178±0,0083) Ω	or	10 000,178 Ω (1±0,83*10^{-6})

Two facts must be taken into consideration when the numerical expression is chosen: First, the measurement uncertainty is based on a probability statement. The value stated should, therefore, not comprise more than two significant digits. In fact, probabilities can be indicated with an accuracy of just 1 to 2%[3]. If more than two significant digits result from the calculation, the value obtained must be mathematically rounded. It must be rounded up whenever a change of the value by more than 5% results. Secondly, the precision of the measured value is to be limited to the most significant digit of the standard uncertainty of measurement [2]. Although the notation 3*) is permissible, it should not be used to state the standard uncertainty of measurement, as it may easily lead to confusion with the expanded uncertainty of measurement.

6 The view of trade and industry: Expanded uncertainty of measurement

The standard uncertainty of measurement is not suited to prove conformity of a measured value with a specification. It is true that it indicates the quality of a measurement result, however, proof of the conformity requires a range which covers a high fraction of values consistent with the measurement conditions. Trade and industry therefore use the *expanded uncertainty of measurement* derived from the standard uncertainty of measurement. It is defined by

$$U = k \cdot u(y) \tag{24}$$

where k is the coverage factor. The coverage factor is chosen so that the uncertainty interval

$$I_Y = \left[y - U; y + U \right] \tag{25}$$

covers a wide fraction of values. The fraction covered is referred to as *coverage probability p.* As in the naive view the present one makes an uncertainty interval available which can be used for comparisons. The advantage is that the calculation via the intermediate step of the standard uncertainty makes use of a method which combines the frequency statistics and the probability of assessment.

The calibration services which are members of the EA, e.g. DKD, have agreed to use a uniform coverage probability of 95% to calculate the coverage factor for the purpose of calibrations. The coverage factor can be determined from the distribution for each coverage probability. The Central Limit Theorem is applied here. As several influence quantities generally play a role in more precise

[3] Statistical statements, such as the probability with which a specific event from among several possible events will occur, can be made with an accuracy of 1% only under the most favourable conditions. For example, in order to determine with the above accuracy whether a dice behaves ideally, i.e. whether the different numbers of its spots appear with the same probability, one would really have to make several thousand throws.

measurements, in good approximation a normal distribution usually results for the values of the measurand. The standard coverage factor $k = 2$ customary in calibrations results for this distribution.

Only the form 3*) is permissible in the statement of the complete measurement result; it is completed by stating of the coverage factor used:

***The measured resistance of the calibrated* 10 kW *standard resistor at a temperature of* 23,00 °C *and a measuring current of* 100 mA is**

(10 000,178±0,017) W

10 000,178 W ±17 mW

or 10 000,178 W (1±1,7*10^{-6})

The uncertainty of measurement stated is obtained by multiplying the standard uncertainty of measurement by the coverage factor $k = 2$. For a normal distribution, it corresponds to a coverage probability of 95%. The standard uncertainty of measurement has been determined in accordance with Guideline EA-4/02.

If the deviations from normal distribution are large, e.g. if an uncertainty contribution dominates in the budget (90% or more) or if the standard uncertainty of measurement associated to an input value has been determined by the type A method of evaluation with a few observations only, another coverage factor has to be chosen. The relevant procedures are given in EA-4/02 [2] and in the examples included in Supplement S2 which is under preparation.

7 Conclusion

It is obvious already now that the GUM procedure has far-reaching effects. The correlations between measurement results occurring when the same standard is used are the subject of current investigations. Knowledge of such correlations is important for the judgement of multilateral comparisons. First successful results in the treatment of hysteresis effects in calibration curves are expected soon. The problems arising here are very complex as far as their conceptual treatment is concerned. The results will have strong impacts on the technology of measuring and test equipment.

8 Acknowledgement

I should like to thank my European colleagues of the EA Uncertainty of Measurement Task Force, the chairman of the PTB's measurement uncertainty committee, Dr. Wolfgang Wöger, and the other members of this committee for many intensive and stimulating discussions which have significantly contributed to my understanding of the significance of the GUM.

10 Literature and national/international recommendations

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Teil 2: ***Begriffe für die Anwendung von Meßgeräten***
Teil 3: ***Auswertung von Messungen einer einzelnen Meßgröße, Meßunsicherheit***
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in: VDI-Berichte 1445, VDE-Verlag (Düsseldorf, 1998), pp 153-166

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[i] At the 139th PTB seminar 'Typical problems of the dissemination of HF measurands' held on May 13, 1998, a computer program package named 'GUM Workbench' was presented which guides the user through the determination of the complete measurement result according to the procedures defined in GUM and EA-4/02/DKD-3 and furnishes a printed report in compliance with the rules, which meets the requirements for quality assurance documentation. More detailed information and demo versions are available to those interested at http://www.metrodata.de or http://www.gum.dk .

[iii] The EA has formerly been known as EAL - European co-operation for Accreditation of Laboratories - and the EA-4/02 as EAL-R2. Organisation schedule, document lists and electronic copies of documents are available at http://www.european-accreditation.org . German translations of most of the EA documents are available as DKD (Deutscher Kalibrierdienst - German Calibration Service) documents. A complete list and further information may be found at http://www.nw-verlag.de .